SAS for ANOVAs and Recovery of Interblock, Intercolumn, and Intergradient Information

by

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ABSTRACT

SAS Proc GLM and Proc Mixed programs are described and presented for obtaining analyses of variance (ANOVAs) and for recovering the information for treatments contained in the random effects for incomplete blocks, rows, columns, and/or gradients sums of squares. An incomplete block design with v = bk = 9 treatments in b = 3 incomplete blocks of size k = 3 in r = 3 complete blocks (replicates) is used to describe the programming and output in some detail. A lattice square designed example with v = bk = 16 treatments in b = 4 rows and k = 4 columns in r = 5 complete blocks is used to describe the programming and output for recovering interrow and intercolumn information. An alternative analysis for this example recovering intergradient information instead of intercolumn is presented also.

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SAS FOR ANOVAS AND RECOVERY OF INTERBLOCK, INTERCOLUMN, AND INTERGRADIENT INFORMATION

INTRODUCTION

Since it appears that statistical analyses using SAS Proc GLM and Proc Mixed (See SAS Inst., Inc., 1989, 1992) to recover the random effects information from a mixed model analysis is not widely known, we present programs and output for three types of analyses. First, an incomplete block designed example is used to illustrate how to obtain the various Type I and Type III sums of squares and how to obtain the intrablock means and the means adjusted for interblock information. The second illustrative example is for a lattice square design, and the third example deals with an alternative analysis for incomplete block or lattice rectangle designed experiments when a random gradient effect is present within an incomplete block or within a row. Programs are presented and described. The output for the programs are presented and described also.

INCOMPLETE BLOCK DESIGN

The example used to illustrate the programming and output for an incomplete block design is Example XI.3 from Federer (1955). There are $v = bk = 9$ treatments in $b = 3$ incomplete blocks of size $k = 3$ in each of $r = 3$ replicates (This design is called a triple lattice.). In the program mode of SAS, a datum is written to each line as given in Table 1 for the 27 observations of Example XI.3. Yield or response Y is given in the first column, the replicate number R is given in the second column, the third column designates the incomplete block B in which the response occurred, and the fourth column designates which treatment is associated with the response. This particular data set was filed under the name fed933.dat. The Proc GLM program used to obtain the sums of squares, mean squares, etc.,the intrablock

means, and the expected value of the blocks (eliminating treatment effects) mean square, is

```
00001   /*This is an incomplete block program in Proc GLM.*/
00002   options ls = 78;
00003   libname dummy '.';
00004   data dummy.fed933;
00005   infile 'fed933.dat';
00006   input Y R B T;
00007   Proc GLM = dummy.fed933;
00008   class R B T;
00009   model Y = R B(R) T;
00010   random R B(R);
00011   lsmeans T;
00012   run;
00013   quit;
```

This program was named fed933.pro. Note that the exact order for the information in Table 1 must be preserved in the "input" statement. The "class" variable cannot be a continuous variable. The "model" statement would be the usual linear model without the plus signs, the general mean, and the error term and contains the response Y, the replicate effect R, the incomplete block nested within complete block or replicate effect B(R), and the treatment effect T. In Proc GLM, all terms in the model are considered to be fixed effects. The "random" statement is used only to obtain the expected value of Type III mean squares for random effects. Treatment means adjusted for interblock information are not available in Proc GLM.

The output for the fed933.pro program using the fed933.dat data set is given in Table 2. The first part of the table lists the replicate numbers, the incomplete block numbers within each complete block, and the treatment numbers. The next part of the output is an ANOVA for which all model effects are pooled together, i.e., 16 = 2 + 6 + 8 = (r - 1) + r(b - 1) + (v - 1) degrees of freedom, and the intrablock error is listed as "Error" which is associated with 10 = rv - v -br + 1 degrees of freedom. The "R-Square" is obtained as Model sum of squares divided by Error sum of squares which is 92.7037037/99.85185185. The "Root MSE" is the square root of the Error mean square (standard deviation). The ANOVA in the third

section contains the replicate (ignoring incomplete block and treatment effects), the blocks within replicate (ignoring treatment effects), and treatment (eliminating all other effects) sums of squares in the Type I part. Note the order of the effects listed in the "model" statement determines the ordering of effects eliminated in a Type I analysis. Reordering the effects in the "model" statement will change the Type I part of the ANOVA. The Type III analysis gives the sums of squares for all effects in the "model" eliminating all other effects. The fourth section contains the expected values of the mean squares for all the effects listed in the "random" statement. The coefficients for the various variance components are those obtained by the method of moments (ANOVA) and are the ones usually presented in statistical textbooks describing the analysis for incomplete block designs. Thus, the expected value for the blocks (eliminating treatment effects) mean square given in Cochran and Cox (1957) and Federer (1955) is $\sigma_\varepsilon^2 + 2\sigma_\beta$, where σ_ε^2 = Var(Error) = intrablock error variance component and σ_β^2 = Var(B(R)) = interblock error variance component. The intrablock means for the v = 9 treatments are given in the fifth section of Table 2.

For Proc Mixed, the program for the fed933.dat data set is

```
00001  /*This is an incomplete block program in Proc Mixed.*/
00002  options ls = 78;
00003  libname dummy '.';
00004  data dummy.fed933;
00005  infile 'fed933.dat';
00006  input Y R B T;
00007  Proc Mixed data = dummy.fed933;
00008  class R B T;
00009  model Y = T;
00010  random R B(R);
00011  lsmeans T;
00012  run;
00013  quit;
```

The changes from the Proc GLM program to obtain the Proc Mixed program are in line 00007 where Proc Mixed is substituted for Proc GLM and in line 00009 where only the fixed effects are retained in

the "model" statement.

The output for the above Proc Mixed program is given in Table 3. The first part is identical to Table 2. Since REML estimates of variance components (See Searle *et al.*, 1992) are being obtained by Proc Mixed, a Newton-Raphson algorithm of iterations used to solve the equations, and the number of iterations needed before the convergence criterion is met is given in the second part of Table 3. The REML estimates of the variance components are given in the column under "Estimate' in the third section of the table. *For this particular example*, the REML and method of moments estimates of the variance components are identical. Note that the iteration procedure stopped at the first one. The fourth section contains the number of observations in the data set, the intrablock error = Variance Estimate" mean square, the standard deviation, and four tests of various kinds. The fifth section contains a test for fixed effects using REML estimates of the variance components. The F-ratio presented here is larger than obtained using Proc GLM, and was for all the examples for which Proc Mixed was used. The last part of the table contains the treatment means adjusted for interblock information using REML estimates of the variance components. Since the estimates for the variance components were identical by the two methods, the adjusted treatment means are those given in Federer (1955).

If it is desired to transform the yield or response Y to, say logs or square roots, this can be done very easily by including a statement immediately following the "input" statement. To transform to natural logarithms, write "LY = LOG(Y);" immediately after the "input" statement. The statement "SY = SQRT(Y);" transforms the Y values to square roots of the responses Y. Numerous other transformations are available in SAS (See SAS Users' Guide.).

Two other SAS statements which are useful are "contrast" and "estimate'. A contrast statement for the difference of treatments 1 and 9 is

```
00012  contrast 'T narrow' T 1 0 0 0 0 0 0 0 -1;
```

results in the difference between the two intrablock means, a sum of squares and mean square for the contrast, an F value and the probability of a larger F. To estimate the difference between the

intrablock means for treatments 1 and 9, one may use the following estimate statement

```
00012   estimate 'T narrow' T 1 0 0 0 0 0 0 0 -1;
```

The difference between intrablock means, Student's t value, the probability of a larger t value, and the standard error of a difference for the two means are given. These statements produce results in Proc GLM but not in Proc Mixed (See last section for a discussion of this.). It was not possible to obtain the contrast or estimate of the difference between two treatment means adjusted for interblock information. In the log mode of Proc Mixed, no error statements were given even though no output was obtained for the above two statements.

LATTICE RECTANGLE DESIGNS

The example used to describe the programming for the lattice rectangle class of designs is the one in Table 12.5 of Cochran and Cox (1957). This is a lattice square design with v = bk = 16 treatments in b = 4 rows and k= 4 columns in each of r = 5 complete blocks. The data are given in Table 4. The formatting is Yield, replicate Rep, Row, Column Col, gradient Grad, and treatment Treat. The Grad values are the independent linear regression values centered for position within a Row. Here they are -3, -1, 1, and 3 since there are four positions in each row. This data file was named lsgr1645.dat. A Proc GLM program for this data set and to obtain the usual ANOVA is

```
00001   ./*This is a Proc GLM program for a lattice square.*/
00002   options ls = 78;
00003   libname dummy '.';
00004   data dummy.lsgr1645;
00005   infile 'lsgr1645.dat';
00006   input Yield Rep Row Col Grad Treat;
00007   Proc GLM data = dummy.lsgr1645;
00008   class Rep Row Col Treat;
00009   model Yield = Rep Row(Rep) Col(Rep) Treat;
00010   random Rep Row(Rep) Col(Rep);
```

```
00011   lsmeans T;
00012   run;
00013   quit;
```

The program was named lasq1645.pro and the output of the program is given in Table 5. The explanation of the output for Proc GLM follows that given for Table 2. Note that all sums of squares agree with those given in Table 12.7 of Cochran and Cox (1957).

The Proc Mixed program for the lattice square analysis of the lsgr1645.dat data set is

```
00001   /*This is a Proc Mixed program for a lattice square.*/
00002   options ls = 78;
00003   libname dummy '.';
00004   data dummy.lsgr1645;
00005   infile 'lsgr1645.dat';
00006   input Yield Rep Row Col Grad Treat;
00007   Proc Mixed data = dummy.lsgr1645;
00008   class Rep Row Col Treat;
00009   model Yield = Treat'
00010   random Rep Row(Rep) Col(Rep);
00011   lsmeans Treat;
00012   run;
00013   quit;
```

The output for the above program is given in Table 6. The description for the output of Table 3 should suffice to explain the output for Table 6. The REML and ANOVA estimates for the variance components are different for this example. They are

	σ_ε^2	σ_ρ^2	σ_γ^2
REML estimate	23.864	10.911	3.169
ANOVA estimate	22.67	15.26	4.88

where these are the interrow-column, row, and column variance components. The F-ratio of treatment to error mean squares in Proc Mixed is 1.71 whereas it was 0.94 for Proc GLM. The difference in the variance component estimates results in different adjusted means from those given in Table 12.6 of Cochran and Cox (1957).

Note that several iterations were needed to obtain the REML estimates of the variance components.

DIFFERENTIAL GRADIENTS IN THE ROWS

An alternative analysis for an incomplete block or lattice rectangle design is to include a term for effects of differential gradients within each incomplete block or within each row of the design. In lattice rectangle designs the column effect is omitted. This analysis is exemplified for the data of Table 4, lsgr1645.dat. The Grad term is used in place of the Col term. This analysis has been described in detail by Federer (1995a, 1995b). The SAS Proc GLM program for obtaining the sums of squares and the expected value of the row (eliminating all other effects) mean square is

```
00001  /*This is a Proc GLM program for differential gradients.*/
00002  options ls = 78;
00003  libname dummy '.';
00004  data dummy.lsgr1645;
00005  infile 'lsgr1645.dat';
00006  input Yield Rep Row Col Grad Treat;
00007  Proc GLM data = dummy.lsgr1645;
00008  class Rep Row Col Treat;
00009  model Yield = Rep Row(Rep) Grad*Row(Rep) Treat;
00010  random Rep Row(Rep);
00011  lsmeans Treat;
00012  run;
00013  quit;
```

Note the manner in which the gradient effect is handled. Grad cannot be a class effect since it is a continuous variable. An effect must be a class effect before it can be a random effect in Proc GLM. Using the notation Grad*Row(Rep) and treating it as a fixed effect results in the correct sums of squares and the correct expected value for the row (eliminating all other effects) mean square (Confirmed using Mathematica.). The coefficient for this variance component is 2.8149. The output of the above program is given in Table 7.

A Proc Mixed program for the analysis of the lsgr1645.dat data set is given below:

```
00001  /*This is a Proc Mixed program for differential gradients.*/
00002  options ls = 78;
00003  libname dummy '.';
00004  data dummy.lsgr1645;
00005  infile 'lsgr1645.dat';
00006  input Yield Rep Row Col Grad Treat;
00007  Proc Mixed data = dummy.lsgr1645;
00008  class Rep Row Col Treat;
00009  model Yield = Treat;
00010  random Rep Row(Rep) Grad*Row(Rep);
00011  lsmeans Treat;
00012  run;
00013  quit;
```

Note the "model" and "random" statements as they differ from Proc GLM. Proc Mixed has several differences from Proc GLM. The intrarow-gradient error variance component is 18.7704 versus 18.97 for Proc GLM. The F-ratio goes from 1.22 for Proc GLM to 1.91 (p value of 0.07) for Proc Mixed. The REML estimate of the component of variance for gradients is 1.398 while the ANOVA estimate is 1.22. The component of variance for rows is 11.257 from Proc Mixed while it is 14.20 by the method of moments. These differences and the use of REML estimates of variance components result in different values for treatment means adjusted for interrow and intergradient information (See Federer, 1995b, and Table 8)

PROC MIXED, ESTIMATE, CONTRAST, AND BLUPS

This section was added owing to information received from Russ Wolfinger, SAS Institute Inc.,. He has shown how to use the "estimate" and "contrast" statements in Proc Mixed and how to obtain various other information such as BLUP estimates of random effects. In the program, both "narrow" and "broad" contrast and estimate statements are included but the ones desired are those from the "broad" statement. For the following program, they gave the same

result but for the program run by Russ Wolfinger they give different results. This writer does not know what the "narrow" ones are, what they mean, or how they are obtained. The program used to obtain the results in Table 9 is:

```
00001  /*This is a program for contrast and estimate statements.*/
00002  options ls = 78;
00003  libname dummy '.';
00004  data dummy.fed933;
00005  infile 'fed933.dat';
00006  input Y R B T;
00007  Proc Mixed data = dummy.fed933;
00008  class R B T;
00009  model Y = T/solution;
00010  random R B(R)/solution;
00011  lsmeans T;
00012  contrast 'T broad' T 1 0 0 0 0 0 0 0 -1;
00013  contrast 'T narrow' T 1 0 0 0 0 0 0 0 -1;
00014  estimate 'T1 broad' intercept 1 T 1 0 0 0 0 0 0 0 0;
00015  estimate 'T1 narrow' intercept 1 T 1 0 0 0 0 0 0 0 0;
00016  run;
00017  quit;
```

The output from this program is given in Table 9. The additional items in this Table which are not in Table 3 are:

solutions for fixed effects,
solutions for random effects,
estimate statement results, broad,
estimate statements, narrow,
contrast statements. broad, and
contrast statements, narrow.

To obtain the fixed effect solutions, the restriction that treatment 9, the last one, is equal to zero was used. This means that the treatment 9 effect obtained under the restriction that the sum of the treatment effects is equal to zero, was subtracted from all other treatment effects . Hence the solutions are differences of treatment effects. This and the fact that ANOVA and REML estimates of the

variance components are equal for this example, are the reasons that the variances of differences, 0.74 and 0.76, are exactly the same as given by Federer (1955). The degrees of freedom associated with each treatment effect is taken by the Proc Mixed procedure to be that for the intrablock error term. This is not correct as the degrees of freedom for this standard error is unknown and must be approximated. These effects have been adjusted for interblock, a random effect, information. The solutions for random effects R and B(R) are the well-known BLUP solutions. The last items in Table 9 are the treatments means adjusted for interblock information and are identical to those given by Federer (1955). Note that the T9 lsmean is identical to the intercept value, resulting from the fact that the T9 effect was set equal to zero.

Russ Wolfinger used the following program statements:

```
data fed;
        input y r b t;
        datalines; (27 observations follow)
run;
```

His program was:

```
proc mixed data = fed;
        class r b t;
        model y = t / solution;
        random r b(r) / solution;
        contrast 'T broad' t 1 0 0 0 0 0 0 0 -1;
        contrast 'T narrow' t 1 0 0 0 0 0 0 0 -1:
                b(R) .33 -.33 0 0 0 0 -.33 0 .33;
        estimate 'T1 broad' intercept 1 t 1 0 0 0 0 0 0 0 0;
        estimate 'T1 narrow' intercept 1 t 1 0 0 0 0 0 0 0 0:
                b(R) .33 0 0 0 .33 0 0 0 .33;
run;
```

The output he obtained from this program is given in Table 10. In the b(R) statement, the value 0.33 = 1 / 3 = 1 / r and appears in the incomplete blocks where treatment 1, T1, occurred. T1 occurred in blocks 12, 22, and 33. T1 and T9 occurred together in block 22 as a difference and hence the block effect summed to zero. Here we see that the T1 broad estimate is identical to the lsmean for T1. The T

value, student's t, 12.78 = 7.21095008 / 0.56402138. The contrast of T1 - T9 = 0.37294686 and the F value is 0.25 = (0.37294686 / 0.73927552)^2, and similarly for the narrow contrast. There is a difference in the values given for the broad and narrow statements but not for the program used to obtain the results given in Table 9..

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Table 1. Data set fed933.dat, Example XI.3, Federer (1955).

```
+PROGRAM EDITOR--------------------------------------------------------------
¦Command ===>
¦NOTE: 27 line(s) included.
¦00001 8 1 1 1
¦00002 5 1 1 7
¦00003 3 1 1 4
¦00004 3 1 2 3
¦00005 2 1 2 6
¦00006 6 1 2 9
¦00007 3 1 3 8
¦00008 7 1 3 5
¦00009 3 1 3 2
¦00010 3 2 1 2
¦00011 3 2 1 7
¦00012 3 2 1 6
¦00013 5 2 2 5
¦00014 7 2 2 9
¦00015 8 2 2 1
¦00016 3 2 3 4
¦00017 2 2 3 8
¦00018 4 2 3 3
¦00019 2 3 1 8
¦00020 2 3 1 7
+-----------------------------------------------------------------ZOOM---------
* gaea                                                                  15:4

+PROGRAM EDITOR--------------------------------------------------------------
¦Command ===>
¦
¦00021 7 3 1 9
¦00022 3 3 2 4
¦00023 3 3 2 5
¦00024 3 3 2 6
¦00025 2 3 3 2
¦00026 4 3 3 3
¦00027 6 3 3 1 ;
```

Table 2. Output from Proc GLM program for fed933.dat.

```
+OUTPUT----------------------------------------------------------------------
¦Command ===>
¦
¦                                The SAS System
¦                                                    15:38 Friday, June 9, 199
¦
¦                          General Linear Models Procedure
¦                             Class Level Information
¦
¦                          Class     Levels     Values
¦
¦                          R              3     1 2 3
¦
¦                          B              3     1 2 3
¦
¦                          T              9     1 2 3 4 5 6 7 8 9
¦
¦                        Number of observations in data set = 27
¦
¦
¦
+----------------------------------------------------------------ZOOM----------
* gaea                                                                15:44:

+OUTPUT----------------------------------------------------------------------
¦Command ===>
¦
¦                                The SAS System
¦                                                    15:38 Friday, June 9, 199
¦
¦                          General Linear Models Procedure
¦
¦Dependent Variable: Y
¦                                        Sum of          Mean
¦Source                      DF         Squares        Square    F Value     Pr >
¦
¦Model                       16     92.70370370    5.79398148       8.11     0.001
¦
¦Error                       10      7.14814815    0.71481481
¦
¦Corrected Total             26     99.85185185
¦
¦                     R-Square           C.V.       Root MSE              Y Mea
¦
¦                     0.928412       20.75238       0.845467            4.07407
¦
¦
+----------------------------------------------------------------ZOOM----------
* gaea                                                                15:45:
```

Table 2 continued.

```
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                    15:38 Friday, June 9, 199

                        General Linear Models Procedure

Dependent Variable: Y

Source                      DF       Type I SS     Mean Square    F Value     Pr >

R                            2      3.85185185      1.92592593       2.69     0.115
B(R)                         6     32.66666667      5.44444444       7.62     0.002
T                            8     56.18518519      7.02314815       9.83     0.000

Source                      DF      Type III SS     Mean Square    F Value     Pr >

R                            2      3.85185185      1.92592593       2.69     0.115
B(R)                         6      7.66666667      1.27777778       1.79     0.198
T                            8     56.18518519      7.02314815       9.83     0.000


+-----------------------------------------------------------------ZOOM-----------
* gaea                                                                  15:46:

+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                    15:38 Friday, June 9, 199

                        General Linear Models Procedure

Source       Type III Expected Mean Square

R            Var(Error) + 3 Var(B(R)) + 9 Var(R)

B(R)         Var(Error) + 2 Var(B(R))

T            Var(Error) + Q(T)


+-----------------------------------------------------------------ZOOM-----------
* gaea                                                                  15:46:
```

Table 2 continued.

```
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 19

                        General Linear Models Procedure
                              Least Squares Means

                                T             Y
                                          LSMEAN

                                1     7.05555556
                                2     2.38888889
                                3     4.27777778
                                4     2.88888889
                                5     4.61111111
                                6     3.22222222
                                7     3.11111111
                                8     2.05555556
                                9     7.05555556


+-----------------------------------------------------------------ZOOM---------
* gaea                                                                   15:4
```

Table 3. Output from Proc Mixed program for fed933.dat.

```
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                    15:38 Friday, June 9, 199

                              The MIXED Procedure

                            Class Level Information

                         Class     Levels  Values

                         R              3  1 2 3
                         B              3  1 2 3
                         T              9  1 2 3 4 5 6 7 8 9

+-----------------------------------------------------------------ZOOM-------
* gaea                                                              15:50:

+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                    15:38 Friday, June 9, 199

                              The MIXED Procedure

                       REML Estimation Iteration History

                  Iteration  Evaluations     Objective     Criterion

                          0            1   28.54212819
                          1            1   27.31174135    0.00000000

                              Convergence criteria met.

+-----------------------------------------------------------------ZOOM-------
```

Table 3 continued.

```
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                      15:38 Friday, June 9, 199

                              The MIXED Procedure

                    Covariance Parameter Estimates (REML)

    Cov Parm            Ratio       Estimate      Std Error        Z  Pr > |Z|

    R             0.05699482     0.04074074     0.24743813     0.16    0.8692
    B(R)          0.39378238     0.28148148     0.40200458     0.70    0.4838
    Residual      1.00000000     0.71481481     0.31967490     2.24    0.0253

+-----------------------------------------------------------------ZOOM----------
* gaea                                                                  15:52:

+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                      15:38 Friday, June 9, 19

                              The MIXED Procedure

                        Model Fitting Information for Y

                    Description                          Value

                    Observations                       27.0000
                    Variance Estimate                   0.7148
                    Standard Deviation Estimate         0.8455
                    REML Log Likelihood               -30.1968
                    Akaike's Information Criterion    -33.1968
                    Schwarz's Bayesian Criterion      -34.5323
                    -2 REML Log Likelihood             60.3935

+-----------------------------------------------------------------ZOOM----------
* gaea                                                                  15:53
```

Table 3 continued.

+OUTPUT----------
Command ===>

The SAS System 1(
15:38 Friday, June 9, 199!

The MIXED Procedure

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
T	8	10	11.69	0.0004

The MIXED Procedure

Least Squares Means

Level	LSMEAN	Std Error	DDF	T	Pr > |T|
T 1	7.21095008	0.56402138	10	12.78	0.0000

+----------ZOOM----------
* gaea 15:53:

+OUTPUT----------
Command ===>

The SAS System 1
15:38 Friday, June 9, 199

The MIXED Procedure

Least Squares Means

Level	LSMEAN	Std Error	DDF	T	Pr > |T|
T 2	2.54428341	0.56402138	10	4.51	0.0011
T 3	3.93590982	0.56402138	10	6.98	0.0000
T 4	2.95104670	0.56402138	10	5.23	0.0004
T 5	4.82866345	0.56402138	10	8.56	0.0000
T 6	2.91143317	0.56402138	10	5.16	0.0004
T 7	3.23542673	0.56402138	10	5.74	0.0002
T 8	2.21095008	0.56402138	10	3.92	0.0029
T 9	6.83800322	0.56402138	10	12.12	0.0000

+----------ZOOM----------

Table 4. Data set lsgr1645.dat, Table 12.5, Cochran and Cox (1957).

```
+PROGRAM EDITOR----------------------------------------------------------------
¦Command ===>
¦NOTE: 80 line(s) included.
¦00001 9.0 1 1 1 -3 10
¦00002 20.3 1 1 2 -1 12
¦00003 17.7 1 1 3  1  9
¦00004 26.3 1 1 4  3 11
¦00005  4.7 1 2 1 -3  2
¦00006  9.0 1 2 2 -1  4
¦00007  7.3 1 2 3  1  1
¦00008  8.3 1 2 4  3  3
¦00009  9.0 1 3 1 -3 14
¦00010  6.7 1 3 2 -1 16
¦00011 11.7 1 3 3  1 13
¦00012  4.3 1 3 4  3 15
¦00013  4.0 1 4 1 -3  6
¦00014  5.0 1 4 2 -1  8
¦00015  5.7 1 4 3  1  5
¦00016 14.3 1 4 4  3  7
¦00017 19.0 2 1 1 -3  5
¦00018  8.7 2 1 2 -1 12
¦00019 13.0 2 1 3  1 15
¦00020 15.7 2 1 4  3  2
+-------------------------------------------------------------------ZOOM---------
* gaea                                                                  16:1

+PROGRAM EDITOR----------------------------------------------------------------
¦Command ===>
¦
¦00021 12.0 2 2 1 -3 10
¦00022  6.0 2 2 2 -1  7
¦00023 15.3 2 2 3  1  4
¦00024 12.0 2 2 4  3 13
¦00025 12.7 2 3 1 -3 16
¦00026  6.3 2 3 2 -1  1
¦00027  1.7 2 3 3  1  6
¦00028 13.0 2 3 4  3 11
¦00029  3.7 2 4 1 -3  3
¦00030  3.7 2 4 2 -1 14
¦00031  8.0 2 4 3  1  9
¦00032 13.3 2 4 4  3  8
¦00033 17.0 3 1 1 -3 10
¦00034  7.0 3 1 2 -1 15
¦00035 10.3 3 1 3  1  8
¦00036  1.3 3 1 4  3  1
¦00037 11.3 3 2 1 -3  9
¦00038 12.3 3 2 2 -1 16
¦00039  3.0 3 2 3  1  7
¦00040  5.3 3 2 4  3  2
+-------------------------------------------------------------------ZOOM---------
* gaea                                                                  16:
```

Table 4 continued.

```
+PROGRAM EDITOR-------------------------------------------------------------
¦Command ===>
¦
¦00041 12.3 3 3 1 -3 12
¦00042  8.7 3 3 2 -1 13
¦00043  8.0 3 3 3  1  6
¦00044  9.3 3 3 4  3  3
¦00045 30.3 3 4 1 -3 11
¦00046 22.3 3 4 2 -1 14
¦00047 11.0 3 4 3  1  5
¦00048 12.7 3 4 4  3  4
¦00049  5.0 4 1 1 -3 16
¦00050 10.3 4 1 2 -1 12
¦00051  5.7 4 1 3  1  8
¦00052 12.7 4 1 4  3  4
¦00053  2.7 4 2 1 -3 11
¦00054  6.7 4 2 2 -1 15
¦00055 10.3 4 2 3  1  3
¦00056  5.7 4 2 4  3  7
¦00057  1.0 4 3 1 -3  1
¦00058 10.3 4 3 2 -1  5
¦00059 11.3 4 3 3  1  9
¦00060 11.7 4 3 4  3 13
+-----------------------------------------------------------------ZOOM--------
* gaea                                                                  16:

+PROGRAM EDITOR-------------------------------------------------------------
¦Command ===>
¦
¦00061 11.0 4 4 1 -3  6
¦00062 19.0 4 4 2 -1  2
¦00063 20.7 4 4 3  1 14
¦00064 29.7 4 4 4  3 10
¦00065  2.0 5 1 1 -3  3
¦00066  5.0 5 1 2 -1 16
¦00067  4.0 5 1 3  1  5
¦00068 13.7 5 1 4  3 10
¦00069  9.3 5 2 1 -3  6
¦00070  1.7 5 2 2 -1  9
¦00071  6.3 5 2 3  1  4
¦00072 12.3 5 2 4  3 15
¦00073 16.7 5 3 1 -3 12
¦00074  4.3 5 3 2 -1  7
¦00075 18.7 5 3 3  1 14
¦00076  8.7 5 3 4  3  1
¦00077 16.7 5 4 1 -3 13
¦00078 30.0 5 4 2 -1  2
¦00079 25.7 5 4 3  1 11
¦00080 14.0 5 4 4  3  8 ;
+-----------------------------------------------------------------ZOOM--------
* gaea                                                                  16:
```

Table 5. Output from Proc GLM program for lsgr1645.dat.

```
+OUTPUT----------------------------------------------------------------------
Command ===>

                                The SAS System                          3
                                                      15:38 Friday, June 9, 199

                        General Linear Models Procedure
                            Class Level Information

          Class     Levels    Values

          R              5    1 2 3 4 5

          ROW            4    1 2 3 4

          COL            4    1 2 3 4

          TREAT         16    1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

                    Number of observations in data set = 80

+-----------------------------------------------------------------ZOOM-----------
* gaea                                                                 16:18:(

+OUTPUT----------------------------------------------------------------------
Command ===>

                                The SAS System                          3
                                                      15:38 Friday, June 9, 199

                        General Linear Models Procedure

Dependent Variable: Y
                                      Sum of          Mean
Source                    DF         Squares        Square    F Value     Pr > I

Model                     49     2928.370083     59.762655       2.64      0.002

Error                     30      680.167917     22.672264

Corrected Total           79     3608.538000

                   R-Square             C.V.      Root MSE              Y Mear

                   0.811511         43.66382      4.761540          10.90500

+-----------------------------------------------------------------ZOOM-----------
```

Table 5 continued.

```
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System                            3
                                                    15:38 Friday, June 9, 199

                        General Linear Models Procedure

Dependent Variable: Y

Source                    DF       Type I SS     Mean Square    F Value      Pr > F

R                          4       31.563000        7.890750       0.35      0.843
ROW(R)                    15     1844.545000      122.969667       5.42      0.000
COL(R)                    15      732.810000       48.854000       2.15      0.035
TREAT                     15      319.452083       21.296806       0.94      0.535

Source                    DF     Type III SS     Mean Square    F Value      Pr > F

R                          4       31.563000        7.890750       0.35      0.843
ROW(R)                    15     1026.755833       68.450389       3.02      0.004
COL(R)                    15      559.589583       37.305972       1.65      0.119

+-----------------------------------------------------------------ZOOM---------
* gaea                                                                16:19:0

+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System                            39
                                                    15:38 Friday, June 9, 1995

                        General Linear Models Procedure

Dependent Variable: Y

Source                    DF     Type III SS     Mean Square    F Value      Pr > F

TREAT                     15      319.452083       21.296806       0.94      0.5350

+-----------------------------------------------------------------ZOOM---------
```

Table 5 continued.

```
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 1

                          General Linear Models Procedure

Source       Type III Expected Mean Square

R             Var(Error) + 4 Var(COL(R)) + 4 Var(ROW(R)) + 16 Var(R)

ROW(R)        Var(Error) + 3 Var(ROW(R))

COL(R)        Var(Error) + 3 Var(COL(R))

TREAT         Var(Error) + Q(TREAT)

+-----------------------------------------------------------------ZOOM---------
* gaea                                                                   16:2

+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 1

                          General Linear Models Procedure
                               Least Squares Means

                                TREAT              Y
                                                LSMEAN

                                1           8.4966667
                                2          13.8966667
                                3           9.5883333
                                4          11.3883333
                                5           8.7633333
                                6           9.3800000
                                7           7.5966667
                                8           8.7300000
                                9           9.2383333
                                10         13.4800000
                                11         16.1133333
                                12         12.0383333
                                13          8.7466667
                                14         14.7216667
                                15          8.9133333
                                16         13.3883333

                                                                ----ZOOM---------
```

Table 6. Output from Proc Mixed program for lsgr1645.dat.

```
+OUTPUT-----
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 19

                                The MIXED Procedure

                              Class Level Information

                Class      Levels  Values

                R               5  1 2 3 4 5
                ROW             4  1 2 3 4
                COL             4  1 2 3 4
                TREAT          16  1 2 3 4 5 6 7 8 9 10 11 12 13
                                   14 15 16

+-------------------------------------------------------------ZOOM---------
* gaea                                                                  16:2

+OUTPUT-----
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 1

                                The MIXED Procedure

                          REML Estimation Iteration History

                Iteration  Evaluations     Objective      Criterion

                        0            1  320.75064929
                        1            3  316.36234199     0.00037412
                        2            1  316.29770396     0.00000930
                        3            1  316.29620730     0.00000001

                            Convergence criteria met.

+-------------------------------------------------------------ZOOM---------
```

Table 6 continued.

```
+OUTPUT-----------------------------------------------------------------
Command ===>

                                The SAS System
                                                    15:38 Friday, June 9, 1

                              The MIXED Procedure

                    Covariance Parameter Estimates (REML)

 Cov Parm          Ratio       Estimate      Std Error        Z  Pr > |Z|

 R           0.00000000     0.00000000              .        .         .
 ROW(R)      0.45722783    10.91105649     6.44537059     1.69    0.0905
 COL(R)      0.13280685     3.16923625     4.23588385     0.75    0.4543
 Residual    1.00000000    23.86350057     6.49142711     3.68    0.0002

+-----------------------------------------------------------------ZOOM---------
* gaea                                                                  16:2

+OUTPUT-----------------------------------------------------------------
Command ===>

                                The SAS System
                                                    15:38 Friday, June 9, 1

                              The MIXED Procedure

                        Model Fitting Information for Y

                  Description                          Value

                  Observations                       80.0000
                  Variance Estimate                  23.8635
                  Standard Deviation Estimate         4.8850
                  REML Log Likelihood              -216.960
                  Akaike's Information Criterion   -220.960
                  Schwarz's Bayesian Criterion     -225.278
                  -2 REML Log Likelihood            433.9203

+-----------------------------------------------------------------ZOOM---------
```

Table 6 continued.

+OUTPUT----------
Command ===>

The SAS System

15:38 Friday, June 9, 19

The MIXED Procedure

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TREAT	15	30	1.71	0.1028

The MIXED Procedure

Least Squares Means

Level	LSMEAN	Std Error	DDF	T	Pr > |T|
TREAT 1	6.08950412	2.56635025	30	2.37	0.0243
TREAT 2	13.73133430	2.56635025	30	5.35	0.0000
TREAT 3	8.49247317	2.56635025	30	3.31	0.0024
TREAT 4	11.34388365	2.56635025	30	4.42	0.0001
TREAT 5	9.56248023	2.56635025	30	3.73	0.0008
TREAT 6	7.29598980	2.56635025	30	2.84	0.0080
TREAT 7	7.29811356	2.56635025	30	2.84	0.0080
TREAT 8	9.41317933	2.56635025	30	3.67	0.0009
TREAT 9	10.11393982	2.56635025	30	3.94	0.0004
TREAT 10	15.18439125	2.56635025	30	5.92	0.0000
TREAT 11	17.90505021	2.56635025	30	6.98	0.0000
TREAT 12	12.84656061	2.56635025	30	5.01	0.0000
TREAT 13	11.03255122	2.56635025	30	4.30	0.0002
TREAT 14	14.25004172	2.56635025	30	5.55	0.0000
TREAT 15	9.29444810	2.56635025	30	3.62	0.0011
TREAT 16	10.62605892	2.56635025	30	4.14	0.0003

----------ZOOM----------

Table 7. Output from Proc GLM program for differential gradients.

```
+OUTPUT----------------------------------------------------------------------
Command ===>
NOTE: At top
                                The SAS System                          5
                                                  15:38 Friday, June 9, 19

                        General Linear Models Procedure
                            Class Level Information

          Class     Levels    Values

          R              5    1 2 3 4 5

          ROW            4    1 2 3 4

          COL            4    1 2 3 4

          TREAT         16    1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

                    Number of observations in data set = 80

+-----------------------------------------------------------ZOOM----------
* gaea                                                          16:33
+OUTPUT----------------------------------------------------------------------
Command ===>
                                The SAS System                          5
                                                  15:38 Friday, June 9, 19

                        General Linear Models Procedure

Dependent Variable: Y
                                    Sum of          Mean
Source                      DF      Squares        Square    F Value     Pr >

Model                       54   3134.276383     58.042155       3.06    0.00

Error                       25    474.261617     18.970465

Corrected Total             79   3608.538000

                  R-Square          C.V.       Root MSE              Y Me

                  0.868572       39.94048       4.355510             10.905

+-----------------------------------------------------------ZOOM----------
```

Table 7 continued.

```
+OUTPUT-----+-------------------------------------------------------------
Command ===>

                                The SAS System
                                                    15:38 Friday, June 9, 19

                          General Linear Models Procedure

Dependent Variable: Y

Source                    DF       Type I SS     Mean Square    F Value      Pr >

R                          4       31.563000        7.890750       0.42      0.79
ROW(R)                    15     1844.545000      122.969667       6.48      0.00
GRAD*ROW(R)               20      910.980000       45.549000       2.40      0.01
TREAT                     15      347.188383       23.145892       1.22      0.32

Source                    DF     Type III SS     Mean Square    F Value      Pr >

R                          4       31.5630000       7.8907500       0.42      0.79
ROW(R)                    15      884.1127443      58.9408496       3.11      0.00
GRAD*ROW(R)               20      765.4958827      38.2747941       2.02      0.04

+-----------------------------------------------------------------ZOOM----------
* gaea                                                                  16:35

+OUTPUT-------------------------------------------------------------------------
Command ===>

                                The SAS System
                                                    15:38 Friday, June 9, 19

                          General Linear Models Procedure

Dependent Variable: Y

Source                    DF     Type III SS     Mean Square    F Value      Pr >

TREAT                     15      347.1883827      23.1458922       1.22      0.32









+-----------------------------------------------------------------ZOOM----------
```

Table 7 continued.

```
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 19

                        General Linear Models Procedure

Source          Type III Expected Mean Square

R               Var(Error) + 4 Var(ROW(R)) + 16 Var(R)

ROW(R)          Var(Error) + 2.8149 Var(ROW(R))

GRAD*ROW(R)     Var(Error) + Q(GRAD*ROW(R))

TREAT           Var(Error) + Q(TREAT)

+-----------------------------------------------------------------ZOOM----------
* gaea                                                                    16:38

+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 19

                        General Linear Models Procedure
                              Least Squares Means

                                TREAT              Y
                                                LSMEAN

                                1           7.0031624
                                2          16.1303818
                                3          11.9046018
                                4          12.9165862
                                5           8.5398621
                                6          10.8440679
                                7           5.2166649
                                8           9.0500145
                                9           7.9690246
                                10         12.9124117
                                11         15.9602515
                                12         12.4141825
                                13          9.4348069
                                14         11.9770655
                                15         11.0053112
                                16         11.2016047
```

Table 8. Output from Proc Mixed program for differential gradients.

```
+OUTPUT----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 19

                            The MIXED Procedure

                          Class Level Information

                Class      Levels  Values

                R               5  1 2 3 4 5
                ROW             4  1 2 3 4
                COL             4  1 2 3 4
                TREAT          16  1 2 3 4 5 6 7 8 9 10 11 12 13
                                   14 15 16
+OUTPUT----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 1

                            The MIXED Procedure

                        REML Estimation Iteration History

              Iteration  Evaluations      Objective      Criterion

                      0            1   320.75064929
                      1            3   313.44983320    0.00067288
                      2            1   313.33214125    0.00002820
                      3            1   313.32759167    0.00000006
                      4            1   313.32758215    0.00000000

                           Convergence criteria met.
+OUTPUT----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 1

                            The MIXED Procedure

                    Covariance Parameter Estimates (REML)

  Cov Parm              Ratio       Estimate     Std Error        Z  Pr > |Z|

  R               0.00000000     0.00000000             .        .         .
  ROW(R)          0.59973661    11.25726995    5.94732994     1.89    0.0584
  GRAD*ROW(R)     0.07445524     1.39755146    0.89873110     1.56    0.1199
  Residual        1.00000000    18.77035654    5.16728433     3.63    0.0003
```

Table 8 continued.

```
* gaea
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 19

                            The MIXED Procedure

                        Model Fitting Information for Y

                  Description                          Value

                  Observations                       80.0000
                  Variance Estimate                  18.7704
                  Standard Deviation Estimate         4.3325
                  REML Log Likelihood              -215.476
                  Akaike's Information Criterion   -219.476
                  Schwarz's Bayesian Criterion     -223.794
                  -2 REML Log Likelihood            430.9517

+-----------------------------------------------------------------ZOOM---------
* gaea                                                                    16:4(
+OUTPUT-----------------------------------------------------------------------
Command ===>

                                The SAS System
                                                        15:38 Friday, June 9, 1

                            The MIXED Procedure

                          Tests of Fixed Effects

                  Source      NDF    DDF   Type III F   Pr > F

                  TREAT        15     25         1.91   0.0736

                            The MIXED Procedure

                            Least Squares Means

      Level                    LSMEAN      Std Error    DDF        T   Pr > |T|

      TREAT 1             5.50387803     2.48063331     25     2.22     0.0358
+-----------------------------------------------------------------ZOOM---------
```

Table 8 continued.

+OUTPUT----------
Command ===>

The MIXED Procedure

Least Squares Means

Level	LSMEAN	Std Error	DDF	T	Pr > |T|
TREAT 2	15.04767639	2.47702021	25	6.07	0.0000
TREAT 3	8.84838625	2.57255200	25	3.44	0.0021
TREAT 4	12.05491597	2.39508859	25	5.03	0.0000
TREAT 5	9.39722793	2.32204120	25	4.05	0.0004
TREAT 6	8.26894654	2.48159219	25	3.33	0.0027
TREAT 7	6.18173650	2.39795747	25	2.58	0.0162
TREAT 8	9.37746781	2.39618712	25	3.91	0.0006
TREAT 9	9.16385288	2.32153157	25	3.95	0.0006
TREAT 10	15.42776142	2.68095394	25	5.75	0.0000
TREAT 11	17.68686255	2.57200068	25	6.88	0.0000

+----------ZOOM----------
* gaea 16:47:3

+OUTPUT----------
Command ===>

The MIXED Procedure

Least Squares Means

Level	LSMEAN	Std Error	DDF	T	Pr > |T|
TREAT 12	13.38207577	2.39799731	25	5.58	0.0000
TREAT 13	10.71280675	2.47872595	25	4.32	0.0002
TREAT 14	13.19930291	2.32128567	25	5.69	0.0000
TREAT 15	10.20507704	2.39541366	25	4.26	0.0003
TREAT 16	10.02202526	2.39560253	25	4.18	0.0003

+----------ZOOM----------

Table 9. Output from Proc Mixed with "estimate" and "contrast" statements.

```
+OUTPUT-----------------------------------------------------------------+
Command ===>
NOTE: At top.
                                The SAS System                        1
                                          10:53 Tuesday, July 18, 1995

                            The MIXED Procedure

                          Class Level Information

                    Class     Levels  Values

                    R              3  1 2 3
                    B              3  1 2 3
                    T              9  1 2 3 4 5 6 7 8 9

+-----------------------------------------------------------ZOOM---------+
~ gaea                                                        11:09:28

+OUTPUT-----------------------------------------------------------------+
Command ===>

                                The SAS System                        2
                                          10:53 Tuesday, July 18, 1995

                            The MIXED Procedure

                       REML Estimation Iteration History

              Iteration  Evaluations     Objective     Criterion

                      0            1   28.54212819
                      1            1   27.31174135    0.00000000

                          Convergence criteria met.

+-----------------------------------------------------------ZOOM---------+
```

Table 9 continued.

```
+OUTPUT----------------------------------------------------------------------+
Command ===>

                                The SAS System                          3
                                                  10:53 Tuesday, July 18, 1995

                            The MIXED Procedure

                    Covariance Parameter Estimates (REML)

    Cov Parm            Ratio       Estimate      Std Error        Z  Pr > |Z|

    R              0.05699482     0.04074074     0.24743813     0.16    0.8692
    B(R)           0.39378238     0.28148148     0.40200458     0.70    0.4838
    Residual       1.00000000     0.71481481     0.31967490     2.24    0.0253

+-----------------------------------------------------------------ZOOM-------+
~ gaea                                                          11:10:57

+OUTPUT----------------------------------------------------------------------+
Command ===>

                                The SAS System                          4
                                                  10:53 Tuesday, July 18, 1995

                            The MIXED Procedure

                      Model Fitting Information for Y

                  Description                          Value

                  Observations                       27.0000
                  Variance Estimate                   0.7148
                  Standard Deviation Estimate         0.8455
                  REML Log Likelihood               -30.1968
                  Akaike's Information Criterion    -33.1968
                  Schwarz's Bayesian Criterion      -34.5323
                  -2 REML Log Likelihood             60.3935

+-----------------------------------------------------------------ZOOM-------+
```

Table 9 continued.

```
+OUTPUT-----------------------------------------------------------------------+
Command ===>

                                The SAS System                          7
                                                  10:53 Tuesday, July 18, 1995

                            The MIXED Procedure

                        Solution for Random Effects

      Parameter        Estimate      Std Error    DDF        T  Pr > |T|

      B(R) 2 3     -0.25182708     0.41397874     10    -0.61     0.5565
      B(R) 3 3     -0.25182708     0.41397874     10    -0.61     0.5565

+-----------------------------------------------------------------ZOOM-------+
~ gaea                                                          11:13:53

+OUTPUT-----------------------------------------------------------------------+
Command ===>

                                The SAS System                          8
                                                  10:53 Tuesday, July 18, 1995

                            The MIXED Procedure

                          Tests of Fixed Effects

                Source       NDF    DDF  Type III F  Pr > F

                T              8     10       11.69  0.0004

                            The MIXED Procedure

                        ESTIMATE Statement Results

   Parameter                  Estimate      Std Error    DDF        T  Pr > |T|

   T1 broad                7.21095008     0.56402138     10    12.78     0.0000
+-----------------------------------------------------------------ZOOM-------+
~
```

Table 9 continued.

11:11:38

+OUTPUT----------
Command ===>

The SAS System 5
10:53 Tuesday, July 18, 1995

The MIXED Procedure

Solution for Fixed Effects

Parameter	Estimate	Std Error	DDF	T	Pr > |T|
INTERCEPT	6.83800322	0.56402138	10	12.12	0.0000
T 1	0.37294686	0.73927552	10	0.50	0.6249
T 2	-4.29371981	0.76257514	10	-5.63	0.0002
T 3	-2.90209340	0.73927552	10	-3.93	0.0028
T 4	-3.88695652	0.76257514	10	-5.10	0.0005
T 5	-2.00933977	0.73927552	10	-2.72	0.0216
T 6	-3.92657005	0.73927552	10	-5.31	0.0003
T 7	-3.60257649	0.73927552	10	-4.87	0.0006
T 8	-4.62705314	0.73927552	10	-6.26	0.0001
T 9	0.00000000	.	.	.	.

----------ZOOM----------
~ gaea

11:12:24

+OUTPUT----------
Command ===>

The SAS System 6
10:53 Tuesday, July 18, 1995

The MIXED Procedure

Solution for Random Effects

Parameter	Estimate	Std Error	DDF	T	Pr > |T|
R 1	0.07051282	0.18859958	10	0.37	0.7163
R 2	0.02820513	0.18859958	10	0.15	0.8841
R 3	-0.09871795	0.18859958	10	-0.52	0.6121
B(R) 1 1	0.43163632	0.41397874	10	1.04	0.3217
B(R) 2 1	-0.52295305	0.41397874	10	-1.26	0.2352
B(R) 3 1	0.57849622	0.41397874	10	1.40	0.1925
B(R) 1 2	0.04048061	0.41397874	10	0.10	0.9240
B(R) 2 2	0.18734052	0.41397874	10	0.45	0.6605
B(R) 3 2	-0.03294934	0.41397874	10	-0.08	0.9381
B(R) 1 3	-0.17839713	0.41397874	10	-0.43	0.6757

----------ZOOM----------
~

Table 9 continued.

```
+OUTPUT-----------------------------------------------------------------+
Command ===>

                                The SAS System                        9
                                                 10:53 Tuesday, July 18, 1995

                              The MIXED Procedure

                           ESTIMATE Statement Results

   Parameter                  Estimate      Std Error    DDF        T  Pr > |T|

   T1 narrow                7.21095008     0.56402138     10    12.78    0.0000

                              The MIXED Procedure

                           CONTRAST Statement Results

              Source                    NDF    DDF        F  Pr > F

              T broad                     1     10     0.25  0.6249
+-----------------------------------------------------------ZOOM-----------+
~ gaea                                                          11:15:21

+OUTPUT-----------------------------------------------------------------+
Command ===>

                                The SAS System                       10
                                                 10:53 Tuesday, July 18, 1995

                              The MIXED Procedure

                           CONTRAST Statement Results

              Source                    NDF    DDF        F  Pr > F

              T narrow                    1     10     0.25  0.6249

                              The MIXED Procedure

                              Least Squares Means

        Level                LSMEAN      Std Error    DDF        T  Pr > |T|

        T 1            7.21095008     0.56402138     10    12.78    0.0000
+-----------------------------------------------------------ZOOM-----------+
```

Table 9 continued.

```
+OUTPUT-----------------------------------------------------------------------+
¦Command ===>
¦                                The SAS System                          11 ¦
¦                                                  10:53 Tuesday, July 18, 1995¦

¦                            The MIXED Procedure

¦                            Least Squares Means

¦       Level              LSMEAN      Std Error    DDF        T  Pr > ¦T¦
¦
¦       T 2           2.54428341     0.56402138     10     4.51     0.0011
¦       T 3           3.93590982     0.56402138     10     6.98     0.0000
¦       T 4           2.95104670     0.56402138     10     5.23     0.0004
¦       T 5           4.82866345     0.56402138     10     8.56     0.0000
¦       T 6           2.91143317     0.56402138     10     5.16     0.0004
¦       T 7           3.23542673     0.56402138     10     5.74     0.0002
¦       T 8           2.21095008     0.56402138     10     3.92     0.0029
¦       T 9           6.83800322     0.56402138     10    12.12     0.0000
¦
¦
+-----------------------------------------------------------------ZOOM-------+
~ gaea                                                            11:16:43

+PROGRAM EDITOR---------------------------------------------------------------+
¦Command ===>
¦
¦00001 /*This is an incomplete block program in Proc GLM.*/
¦00002 options ls = 78;
¦00003 libname  dummy '.';
¦00004 data dummy.fed933;
¦00005 infile 'fed933.dat';
¦00006 input Y R B T;
¦00007 Proc Mixed data=dummy.fed933;
¦00008 class R B T;
¦00009 model Y = T/solution;
¦00010 random R B(R)/solution;
¦00011 lsmeans T;
¦00012 contrast 'T broad' T 1 0 0 0 0 0 0 0 -1;
¦00013 contrast 'T narrow' T 1 0 0 0 0 0 0 0  -1;
¦00014 estimate 'T1 broad' intercept 1 T 1 0 0 0 0 0 0 0 0;
¦00015 estimate 'T1 narrow' intercept 1 T 1 0 0 0 0 0 0 0 0;
¦00016 run;
¦00017 quit;
¦00018
¦00019
¦00020
+-----------------------------------------------------------------ZOOM-------+
~ gaea                                                            11:19:16
```

Table 10. Wolfinger's output for fed933.dat.

The MIXED Procedure

Class Level Information

Class	Levels	Values
R	3	1 2 3
B	3	1 2 3
T	9	1 2 3 4 5 6 7 8 9

REML Estimation Iteration History

Iteration	Evaluations	Objective	Criterion
0	1	28.54212819	
1	1	27.31174135	0.00000000

Convergence criteria met.

Covariance Parameter Estimates (REML)

Cov Parm	Ratio	Estimate	Std Error	Z
R	0.05699482	0.04074074	0.24743813	0.16
B(R)	0.39378238	0.28148148	0.40200458	0.70
Residual	1.00000000	0.71481481	0.31967490	2.24

Covariance Parameter Estimates (REML)

Pr > |Z|
0.8692
0.4838
0.0253

Model Fitting Information for Y

Description	Value
Observations	27.0000
Variance Estimate	0.7148
Standard Deviation Estimate	0.8455
REML Log Likelihood	-30.1968
Akaike's Information Criterion	-33.1968
Schwarz's Bayesian Criterion	-34.5323
-2 REML Log Likelihood	60.3935

Table 10 continued.

Solution for Fixed Effects

Parameter	Estimate	Std Error	DDF	T	Pr > |T|
INTERCEPT	6.83800322	0.56402138	2	12.12	0.0067
T 1	0.37294686	0.73927552	10	0.50	0.6249
T 2	-4.29371981	0.76257514	10	-5.63	0.0002
T 3	-2.90209340	0.73927552	10	-3.93	0.0028
T 4	-3.88695652	0.76257514	10	-5.10	0.0005
T 5	-2.00933977	0.73927552	10	-2.72	0.0216
T 6	-3.92657005	0.73927552	10	-5.31	0.0003
T 7	-3.60257649	0.73927552	10	-4.87	0.0006
T 8	-4.62705314	0.73927552	10	-6.26	0.0001
T 9	0.00000000	.	.	.	.

Solution for Random Effects

Parameter	Estimate	SE Pred	DDF	T	Pr > |T|
R 1	0.07051282	0.18859958	10	0.37	0.7163
R 2	0.02820513	0.18859958	10	0.15	0.8841
R 3	-0.09871795	0.18859958	10	-0.52	0.6121
B(R) 1 1	0.43163632	0.41397874	10	1.04	0.3217
B(R) 2 1	-0.52295305	0.41397874	10	-1.26	0.2352
B(R) 3 1	0.57849622	0.41397874	10	1.40	0.1925
B(R) 1 2	0.04048061	0.41397874	10	0.10	0.9240
B(R) 2 2	0.18734052	0.41397874	10	0.45	0.6605
B(R) 3 2	-0.03294934	0.41397874	10	-0.08	0.9381
B(R) 1 3	-0.17839713	0.41397874	10	-0.43	0.6757
B(R) 2 3	-0.25182708	0.41397874	10	-0.61	0.5565
B(R) 3 3	-0.25182708	0.41397874	10	-0.61	0.5565

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
T	8	10	11.69	0.0004

ESTIMATE Statement Results

Parameter	Estimate	Std Error	DDF	T
T1 broad	7.21095008	0.56402138	10	12.78
T1 narrow	7.33210950	0.50185504	10	14.61

Table 10 continued.

ESTIMATE Statement Results

Pr > |T|
0.0001
0.0001

CONTRAST Statement Results

Source	NDF	DDF	F	Pr > F
T broad	1	10	0.25	0.6249
T narrow	1	10	0.92	0.3590

Least Squares Means

Level	LSMEAN	Std Error	DDF	T	Pr > |T|
T 1	7.21095008	0.56402138	10	12.78	0.0001
T 2	2.54428341	0.56402138	10	4.51	0.0011
T 3	3.93590982	0.56402138	10	6.98	0.0001
T 4	2.95104670	0.56402138	10	5.23	0.0004
T 5	4.82866345	0.56402138	10	8.56	0.0001
T 6	2.91143317	0.56402138	10	5.16	0.0004
T 7	3.23542673	0.56402138	10	5.74	0.0002
T 8	2.21095008	0.56402138	10	3.92	0.0029
T 9	6.83800322	0.56402138	10	12.12	0.0001